Mathematics of Music

Spring 2021

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Overview and objectives: In this interdisciplinary course, we will explore some of the connections between mathematics and music. We will do this on different levels. First, we will see how fundamental concepts in music (for example, rhythm, time signature, scales, keys, intervals, intonation, tuning and symmetry) are based on mathematical principles (geometric series, least common multiples, sine functions, rational numbers, irrational numbers and group theory, to name but a few). On a physical level, we will study the science of sound and the mathematics underlying sound waves and pitch. Lastly, we will investigate how some composers have based their creations on mathematical concepts. Examples include Schoenberg and his twelve-tone music and modern composer Xenakis, who has used computers and probability theory to create "stochastic music".

Apart from studying these connections in class, you will also get to explore them in practice. As part of the course, you will attend at least two local music productions or recitals and report on the mathematical connections you observed. You will also get to incorporate some of these mathematical connections in your own composition, which will serve as your final project in this course.

Required resources:

1. *From Music to Mathematics: Exploring the Connections*, by Gareth E. Roberts.
2. *Music and Mathematics: From Pythagoras to Fractals*, edited by John Fauvel, Raymond Flood and Robin Wilson.
3. *Music: A Mathematical Offering*, by Dave Benson.
   Published online by the author at https://homepages.abdn.ac.uk/d.j.benson/pages/html/maths-music.html.
4. Staff paper.

Course learning outcomes: Formally, the course objectives described above can be summarized in the following course learning outcomes (CLO's):

	CLO	**Instructional Activity**	**Assessment**
(i)	demonstrate an understanding of the multiple connections between mathematics and music;	lecture group activities listening activities reading	class participation homework assignments tests final project
(ii)	develop an understanding of music theory and a deeper appreciation for music;	lecture group activities listening activities reading concert attendances	class participation homework assignments tests final project concert reports
(iii)	develop skills in analytical thinking, critical thinking and abstract reasoning;	lecture group activities reading concert attendances	class participation homework assignments tests concert reports
(iv)	integrate their artistic and analytical skills;	group activities listening activities	class participation homework assignments final project
(v)	reflect upon the Triune God, the Creator of music and mathematics, and their identities as followers of Christ.	lecture group activities	class participation

Program learning outcomes: The course learning outcomes described above reflect the program learning outcomes formulated by the Mathematics department at Westmont College:

(1) Core Knowledge: Students will demonstrate knowledge of the main concepts, skills, and facts of the discipline of mathematics – reflected in CLO (i).

(2) Communication: Students will be able to communicate mathematical ideas following the standard conventions of writing or speaking in the discipline – reflected in CLO (iii).

(3) Creativity: Students will demonstrate the ability to formulate and make progress toward solving non-routine problems – reflected in CLO (iv).

(4) Christian Connection: Students will incorporate their mathematical skills and knowledge into their thinking about their vocations as followers of Christ – reflected in CLO (v).

General education: This course fulfills the *Common Skills: Quantitative and Analytical Reasoning* (QAR) requirement because it emphasizes the ability to interpret, evaluate and communicate quantitative ideas present in musical structures (CLO (i), (iii) and (iv), with the broader description, including specific mathematical topics that will be studied, under **Overview and Objectives**.). Students will apply relevant mathematical and logical methods to analyze sound and concepts in music theory effectively and be able to utilize the results appropriately when making decisions in this field.

This course also fulfills *Reasoning Abstractly* (RA) because it focuses on critical and analytical reasoning about non-empirical, abstract concepts and objects, specifically as it applies to musical structures. Students will learn to understand and evaluate abstract arguments and explanations, analyze abstract concepts and solve abstract problems in music theory and composition (CLO (iii) and (ii) and the broader

description under **Overview and Objectives**).

Students completing this course will be able to:

- make use of mathematical models for studying sound and music theory (QAR) (CLO (i), (iii));
- reflect on the strengths and weaknesses of particular quantitative models or methods as tools in the study of sound and music theory (QAR) (CLO (i), (iii));
- be able to interpret, reflect on, and use quantitative models and data in music composition (QAR) (CLO (i), (iii), (iv));
- identify instances of abstract reasoning about abstract concepts in music theory (in the form of arguments, explanations, proofs, analyses, modeling, or processes of problem solving) (RA) (CLO (iii));
- construct an instance of valid reasoning about abstract concepts in music theory (in the form of arguments, explanations, proofs, analyses, modeling, or processes of problem solving) (RA) (CLO (iii));
- distinguish valid forms of reasoning about abstract concepts in music theory (in the form of arguments, explanations, proofs, analyses, modeling, or processes of problem solving) from invalid and/or fallacious forms of reasoning (RA) (CLO (iii)).

Homework: Homework assignments will be assigned on a regular basis (7-8 assignments during the couse of the semester). The work sheet will be posted on Canvas, typically one week before the due date. You are expected to write your solutions clearly, with logical arguments and proper notation (CLO (iii)). You are more than welcome to collaborate with your class mates on these assignments, as long as you write and turn in your own set of solutions and list the names of your collaborators on your solutions. Late homework will not be accepted without warning – if you cannot make it to class to submit your homework, please contact me before hand so that we can make alternative arrangements.

Concert reports: You are required to attend two musical performances during the semester and turn in a typed 1-2 page report. The purpose of these reports is to observe connections to the course material, to enhance your musical appreciation, and to support your fellow students and the arts (CLO (ii) and (v)). Each review is due within two weeks of the concert.

Tests: We will have two written tests in class during the semester (CLO (i)-(iii)). If you miss a test without a valid excuse, a grade of zero will be recorded. If you need to miss a test and you have a valid excuse (for example, an illness or participating in Westmont sports events), please contact me as soon as possible to arrange a make-up. Test solutions will be posted on Canvas after all students have completed the exam.

Final project: As a conclusion of the course, you will complete a final project consisting of a musical composition and performance demonstrating some of the mathematical concepts you have learned in the course (CLO (i), (iv) and (v)). You will also write a short report to explain the mathematical connections and rationale in your work. Performances of the final project will take place during the scheduled final exam period. The last couple of class periods will be allocated for you to work on your project and to receive constructive criticism.

Grading: Your grade will be calculated as follows:

Class participation:	10%
Homework assignments:	20%
Two concert reports:	5% each
Two tests:	20% each
Final project:	20%

I will assign grades on the usual 90/80/70/60 scale; plus and minus grades will be assigned as appropriate. In borderline cases, I reserve the right to take into account consistency of attendance and participation.

Tentative schedule:

Week 1	Introduction
	Rhythm (Roberts Chapter 1)
Week 2	Rhythm (Roberts Chapter 1)
	Basic Music Theory (Roberts Chapter 2)
Week 3	Basic Music Theory (Roberts Chapter 2)
Week 4	The Science of Sound (Roberts Chapter 3, Benson Chapters 1, 3)
Week 5	The Science of Sound (Chapter 3, Benson Chapters 1, 3)
	Test 1
Week 6	Tuning and Temperament (Roberts Chapter 4)
Week 7	Tuning and Temperament (Roberts Chapter 4)
Week 8	Musical Group Theory (Roberts Chapter 5)
Week 9	Musical Group Theory (Roberts Chapter 5)
Week 10	Change (Bell) Ringing (Roberts Chapter 6, Fauvel Chapter 7)
Week 11	Test 2
	Twelve-Tone Music and Serialism (Roberts Chapter 7)
Week 12	Twelve-Tone Music and Serialism (Roberts Chapter 7)
	Mathematical Modern Music (Roberts Chapter 8, Fauvel Chapter 8)
Week 13	Mathematical Modern Music (Roberts Chapter 8, Fauvel Chapter 8)
Week 14	Final Project: Mathematical Composition
Week 15	Final Project: Mathematical Composition

Technology:

- Schedules and other notices will be posted on Canvas https://westmont.instructure.com/.
- I will be using Canvas to post announcements, so please make sure that you receive Canvas notifications about announcements right away.
- During class, I expect you to be an active participant. Therefore, feel free to use your laptop, tablet, phone or calculator for note-taking or calculations, but make sure that it does not distract you or your classmates. I reserve the right to take away this privilege if it becomes a problem. ☹
- No electronic technology of any kind will be required or allowed during the exams.

Office hours: As listed above, my scheduled office hours are .... Office hours are times that I have reserved to be in my office, available to help you. You do not have to check with me beforehand; just drop in! To get the most out of a visit, I encourage you to come prepared – the clearer you are about what you understand and what you don't understand, the better I can try to help you. Also, never feel stupid for asking questions; that's how you learn and make progress!

Contacting me: If you cannot reach me in person, the best way to contact me is via email (as listed above, my email address is mvanderwalt@westmont.edu). I typically respond quickly during working hours.

Connecting with professors: You are encouraged to take advantage of the *Take a Professor to Lunch Program* as an opportunity to get to know each of your professors over a shared meal. Feel free to contact me about this! ☺ I'm also always happy to talk over a cup of tea in my office (cookies included); you're welcome to pop in if you see me in my office or send me an email if you want to arrange something beforehand.

Attendance: If you miss a significant number of classes, you will almost certainly do poorly in this class. If you miss more than six classes without a valid excuse, I reserve the right to terminate you from the course with a grade of F – this is in line with Westmont's attendance policy, which is available at http://www.westmont.edu/_offices/registrar/academic_policies/attendance-policies.html. Students are responsible for obtaining information and assignments distributed during missed classes. Class notes for missed days should be obtained from a fellow student and not the instructor.

Academic integrity: Dishonesty of any kind may result in loss of credit for the work involved and the filing of a report with the Provost's Office. Major or repeated infractions may result in dismissal from the course with a grade of F. Westmont's plagiarism policy is available at https://westmont.edu/_offices/provost/Plagiarism/policydoc.pdf.

Accommodation procedure: Students who have been diagnosed with a disability (learning, physical or psychological) are strongly encouraged to contact the Disability Services office as early as possible to discuss appropriate accommodations for this course. Formal accommodations will only be granted for students whose disabilities have been verified by the Disability Services office. These accommodations may be necessary to ensure your equal access to this course. Please contact Sheri Noble, Director of Disability Services (310A Voskuyl Library, 805-565-6186, snoble@westmont.edu) or visit http://www.westmont.edu/_offices/disability/.

Final comments: Lastly, I really want you to succeed in this course and to enjoy mathematics and music as much as I do! Please don't hesitate to contact me about anything.

> *There is geometry in the humming of the strings, there is music in the spacing of the spheres.*
> *– Pythagoras*